**NUMERICAL APPROXIMATIONS**

**James Rosado**

Let us begin with the following two polynomial approximations to a function :

If we let then and substitute into the two equations above, keep in mind that these approximations are fairly accurate when is close to *x*.

We can calculate the remainders and errors with these approximations using Taylors Theorem, where and correspond to the remainders and errors of the approximations.

Where , , and , or . Now take equation (3) and isolate the derivative part to get,

If we replace *h* with –*h* in (5) we get,

Equation (5) is called the **FORWARD** difference approximation and (6) is called the **BACKWARD** difference approximation. If we add equations (5) and (6) we get the **CENTERED** difference approximation:

Therefore,

Suppose now we consider the polynomial for *n* = 3,

If we subtract the (9) from (8) we get,

If we isolate , we obtain an approximation to the derivative at a given point.

Now let us find the second order approximation by adding equations (8) and (9),

Isolate the second derivative term to get,

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**SUMMARY:**

F] Forwd-Difference-Approx. 🡺

B] Backwd-Difference-Approx. 🡺

C] Centered-Diff.-Approx. 🡺

D] Second-Order Approx. 🡺

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Let us now consider a numerical approximation to the heat equation. Recall,

With boundary conditions: and initial condition:

Let us apply the forward difference approximation to the derivative on *t*, we will let *h* = :

And let us find the second-order approximation on *x*:

If we take the two equation above and substitute their right sides into the heat equation we get,

Therefore,

Where,

We have now found a relation that uses the start position , start time , next time , and next position . So we will make the following substitutions into (11):

Now (11) becomes,

If we consider the short notation, then the above line becomes,

Since our derivation is based on the 1 dimensional case of a rod where *L* is the length of the rod and *N* is the number of partitions we make. And our samples will be indexed from and our time samples will be indexed from